Model Checking Communicating Processes: Run Graphs, Graph Grammars & MSO

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Agenda

- formal model of distributed recursive communicating processes
- recall un-/decidability of basic verification questions
- show line of attack via under-approximative approaches

- from semantic “boundedness” to graph grammars
- and back to decidability of MSO model checking

article takes different way:
- try to extend known results beyond reachability
- directly arrive at undecidability
- extend proof-idea of reachability result for subclass of RCPS
- arrive at graph grammars for run graphs
- apply known HRG-treewidth-MSO connection
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# Simple Reverse Echo Server

```python
import socket

def send_rev_log(conn, l):
    if not l:
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    else:
        conn.send(l[-1])
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HOST = ''
PORT = 54321
s = socket.socket(socket.AF_INET, socket.SOCK_STREAM)
s.bind((HOST, PORT))
s.listen(1)

conn, addr = s.accept()

log=[] # local pushdown store

while 1:
    msg = conn.recv(1024)
    if not msg: break

    if msg.startswith("show"):  
        log=send_rev_log(conn,log)
    else:
        log.append(msg)

conn.close()
```

```python
from mpi4py import MPI

comm = MPI.COMM_WORLD
rank = comm.Get_rank()
assert comm.Get_size() == 7

if rank == 0:  # process 0
    data = get_input()
    comm.send(data[:2], dest=1)
    comm.send(data[2:], dest=2)
    res1=comm.recv(source=1)
    res2=comm.recv(source=2)
    result = max(res2,res1)
    assert True

elif rank == 1:  # process 1
    data=comm.recv(source=0)
    comm.send(data[:1], dest=4)
    comm.send(data[1:], dest=6)
    res1=comm.recv(source=4)
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def gcd(x,y):
    if y==0: return x
    else: return gcd(y,x%y)

result = gcd(res1,res2)
comm.send(result,dest=0)
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#...
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---

1. **Sockets-based Client-Server**

- **C** (Client)
- **S** (Server)

2. **MPI-based Master-Worker**

- **M** (Master)
- **W** (Worker)
- **W’** (Worker)
# Verification of Distributed Systems

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## MPI Implementation

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## Concurrency

- The code snippet shows a simple reverse echo server with log manipulation.
- MPI is used to distribute and synchronize data across processes.
- The `send_rev_log` function is recursively called to process a list of messages.
- MPI processes 0 and 1 exchange data and compute a maximum value.
- The `gcd` function calculates the greatest common divisor of two numbers.
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**Verification of Distributed Systems**

- **Concurrency**
- **Asynchronous Communication**
- **Recursion**
Verification of Distributed Systems

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corrience
asynchronous communication
recursion

check safety
Verification of Distributed Systems

check specification in formal logic

- e.g., the server returns all received messages in reverse order

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```

concurrency

asynchronous communication

recursion
Formal Model: RCPS

Formalize by Recursive Communicating Processes

- reliable, asynchronous, unbounded channels $\rightarrow$ fifo queues
- network $\rightarrow$ (communication) graph / architecture
- recursion $\rightarrow$ pushdown stack
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Reachability Verification

\[\Rightarrow \text{reachability undecidable for finite-state RCPS} \]

already \(\mathbb{p} \bowtie\) & \(\overrightarrow{p \leftrightarrow q}\) are Turing powerfull

\(\ni\) need to avoid cyclic communication: impractical 😞

\(\Rightarrow\) and for simple non-cyclic architectures with pushdowns

already for \(\mathbb{p} \overrightarrow{} \mathbb{q}\) : PCP/intersect. of cf langs

\(\ni\) avoid any communication ? 😞 😞 😞
Reachability Verification

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already $p \circlearrowleft$ & $p \xrightarrow{} q$ are Turing powerfull

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- avoid any communication?
Under-Approximation

Idea:

Restrict focus only on subset of possible runs that have a certain form (under-approximate behaviour).

- try to find restriction on runs that have

  (i) good theoretical properties (decidable reachability etc.)

  (ii) is "applicable" in practical domain

  (iii) can decide (syntactically) if restriction holds

  (iv) algorithms can be implemented
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Runs of RCPS

- Interleaving semantics \((run = \text{altern. seq. of states/actions})\)

- Event-based/partial-order semantics \((\text{run graph})\)

\[
\text{init} \xrightarrow{} \text{push a} \xrightarrow{} \text{!m} \xrightarrow{} \text{push b} \xrightarrow{} \text{pop b} \xrightarrow{} ?n \xrightarrow{} \text{pop a}
\]

\[
\text{init} \xrightarrow{} \text{push m} \xrightarrow{} ?m \xrightarrow{} \text{pop m} \xrightarrow{} \text{!n} \xrightarrow{} \text{push m}
\]
Runs of RCPS

- Interleaving semantics (run = altern. seq. of states/actions)

\[ s_0, push\ a, s_2, !m, s_3, push\ b, s_4, push\ m, s_5, ?m, s_6, pop\ b, s_7, \ldots \]
\[ \ldots, s_7, pop\ m, s_8, !n, s_9, ?n, s_{10}, push\ m, s_{11}, pop\ a, s_{12} \]

- Event-based/partial-order semantics (run graph)
Runs of RCPS

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- **event-based/partial-order semantics** (run graph)
Boundedness Conditions: Interleaving

- “ressource”-boundedness
  - atomic communication
  - untangle pushdown and communication actions
  - architecture: tree-like, acyclic,…
  - mixed: half-duplex,…

- family of phase-boundedness restrictions:
  e.g., contexts [Qadeer/Rehof’05]

Reachability decible in $\sim (2)\text{ExpTime}$ / find “shallow” errors

Rarely decidability results beyond reachability
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可达性可解在\(\sim(2)\)ExpTime / 找到“浅”错误

很少 decidability results beyond reachability…
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Boundedness: Partial-Order Case

- bounded MSC (= bounded finite-state RCPS)
  - fix size of channels a priori: finite system
    - semantically fix channel usage:
      for all prefixes of any run the number of unreceived messages on each channel is less than or equal to $b$ (for $b \in \mathbb{N}$)
  - allows to decompose MSC into “building blocks”
  - LTL/MSO model checking decidable
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How to extend MSO model checking ideas to RCPS?

Derive more general notion of boundedness?
How to extend MSO model checking ideas to RCPS?

Goal: proof one, get one for free!

Derive more general notion of boundedness?
Again: Undecidability for finite RCPS

**Theorem [Seese ’75]:**

Let $C_G$ be a class of graphs such that for every integer $k > 1$ there is a graph $G \in C_G$ such that $G$ has the $k \times k$ grid as induced subgraph. Then MSO is undecidable on $C_G$.

MSO undecidable on rungraphs of finite-state RCPS as one can embedd the $k \times k$ grid already on $p \leftrightarrow q$, e.g., for $k = 3$:

\[ G_{3\times3} = 1 \to 2 \to 3 \\
\downarrow \quad \downarrow \quad \downarrow 
\quad 4 \to 5 \to 6 \\
\downarrow \quad \downarrow \quad \downarrow 
\quad 7 \to 8 \to 9 \]

resembles Turing completeness proof [Brand/Zafiropoulo ’83]

\(^1\text{induced subgraph can be extended to minor+induced subgraph.}\)
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MSO undecidable on rungraphs of finite-state RCPS as one can embedd the $k \times k$ grid already on $\text{rungraph}$, e.g., for $k = 3$

$G_{3\times3} = \begin{align*}
1 &\rightarrow 2 \rightarrow 3 \\
4 &\rightarrow 5 \rightarrow 6 \\
7 &\rightarrow 8 \rightarrow 9
\end{align*}$

\[ \text{rungraph} \]

\[ \text{resembles Turing completeness proof [Brand/Zafiropoulo ’83]} \]

\[ ^1 \text{induced subgraph can be extended to minor+induced subgraph.} \]
Again: Undecidability for finite RCPS

Theorem [Seese ’75]:

Let $C_G$ be a class of graphs such that for every integer $k > 1$ there is a graph $G \in C_G$ such that $G$ has the $k \times k$ grid as induced subgraph. Then MSO is undecidable on $C_G$.

\[ G_{3 \times 3} = \begin{align*}
1 & \rightarrow 2 \rightarrow 3 \\
4 & \rightarrow 5 \rightarrow 6 \\
7 & \rightarrow 8 \rightarrow 9
\end{align*} \]

MSO undecidable on rungraphs of finite-state RCPS as one can embed the $k \times k$ grid already on $p \leftrightarrow q$, e.g., for $k = 3$

\[ \cdots 1 \rightarrow 2 \rightarrow 3 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow \cdots \]

resembles Turing completeness proof [Brand/Zafiropoulo ’83]

\[ ^1 \text{induced subgraph can be extended to minor+induced subgraph.} \]
Rungraph of bounded finite-state RCPS

Áreminds of derivations of (context-free) graph grammars
Rungraph of bounded finite-state RCPS
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Rungraph of bounded finite-state RCPS

\[\text{fst}/s^0 \quad 0/0 \quad \text{fst}/q^0 \quad 1/0 \quad \text{fst}/q^0\]

\[\text{s}^0, !, s\]

\[\text{lst}/s^0 \quad \text{lst}/q^0\]

This reminds of derivations of (context-free) graph grammars.
Rungraph of bounded finite-state RCPS

![Diagram of bounded finite-state RCPS rungraph]
Rungraph of bounded finite-state RCPS
Rungraph of bounded finite-state RCPS

reminds of derivations of (context-free) graph grammars 😊
From HRG to MSO Decidability

Theorem [Courcelle ’12] :

Given an HRG that describes a class of graphs $C_G$, and an MSO formula $\varphi$, then checking whether there exists a graph in $C_G$ that satisfies $\varphi$ is decidable.
A HRG for bounded finite RCPS

\[ n^* = \begin{array}{c} 2/2 \\ \end{array} \quad R : \quad \begin{array}{c} 2/2 \\ \end{array} \rightarrow \quad \begin{array}{c} 1/2 \\ \end{array} \quad \begin{array}{c} 2/1 \\ \end{array} \quad \begin{array}{c} 0/2 \\ \end{array} \quad \begin{array}{c} 2/2 \\ \end{array} \quad \begin{array}{c} 1/1 \\ \end{array} \]

non-terminals encode remaining queue capacity

rhs: what can happen next
A HRG for bounded finite RCPS

$n^* = \frac{2}{2}$

$\mathcal{R} : \frac{2}{2} \rightarrow \frac{1}{2}$

non-terminals encode remaining queue capacity

rhs: what can happen next
A HRG for bounded finite RCPS

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\text{2/2}
\end{array}
\end{array} \quad R : \quad \begin{array}{c}
\begin{array}{c}
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\end{array} \quad \rightarrow \quad \begin{array}{c}
\begin{array}{c}
\text{1/2}
\end{array}
\end{array} \quad \rightarrow \quad \begin{array}{c}
\begin{array}{c}
\text{0/2}
\end{array}
\end{array} \quad \rightarrow \quad \begin{array}{c}
\begin{array}{c}
\text{2/2}
\end{array}
\end{array}
\end{array} \quad \rightarrow \quad \begin{array}{c}
\begin{array}{c}
\text{1/1}
\end{array}
\end{array}
\]

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non-terminals encode remaining queue capacity

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\[ n^* = 2/2 \quad R : \quad 2/2 \rightarrow 1/2 \quad 2/2 \rightarrow 1/2 \quad 2/1 \quad 0/2 \quad 2/2 \quad 1/1 \]

- **non-terminals** encode remaining queue capacity
- **rhs:** what can happen next
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\[ n^* = \begin{array}{c} 2/2 \\ \end{array} \quad \mathcal{R} : \begin{array}{c} 2/2 \\ \end{array} \rightarrow \begin{array}{c} 1/2 \\ \end{array} \rightarrow \begin{array}{c} 0/2 \\ \end{array} \rightarrow \begin{array}{c} 2/2 \\ \end{array} \rightarrow \begin{array}{c} 1/1 \\ \end{array} \]

non-terminals encode remaining queue capacity

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\[ n^* = \begin{array}{c} \bullet \ 
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- HRG generates rungraphs of 2-bounded finite RCPS of 2 processes
- generalize: \( p \) processes and bound \( k \)
- rulewidth of run graphs of bounded finite RCPS is in \( \mathcal{O}(p \cdot k) \)
From HRG to MSO Decidability

**Proposition:**

The MSO model checking problem is decidable for rungraphs of bounded finite-state RCPS.

каз already known [Madhusudan’03] [Genest et al.’07], but here new “style” of proof 😊

каз by connection of HRG rulewidth with treewidth:

**Corollary:**

MSC of bounded finite-state RCPS have bounded treewidth.

каз bounded treewidth more general notion of semantic boundedness?
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Undecidability for General RCPS

- reachability (and MSO) undecidable for RCPS

  idea: infinite-grid-as-minor argument!

- look at 2PDS (can reduce RCPS to this case)

\[
G_{3 \times 3} = \begin{array}{c}
1 \rightarrow 2 \rightarrow 3 \\
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\end{array}
\]

\[
G_{3 \times 3}
\]
Reachability is decidable for RCPS iff they are well-queueing, eager, and imbalanced. (even ExpTime-complete!)

original proof:
commutative reordering of interleaving allows 1-stack simulation
reinterpret in partial-order/rungraph setting: decomposition
Theorem [Heußner et al. 2010]:
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⇒ original proof:
  commutative reordering of interleaving allows 1-stack simulation
⇒ reinterpret in partial-order/rungraph setting: decomposition

use pushdown without restriction

atomic

either use pushdown or channels
Ressource Bound $\leftrightarrow$ Grammar Bound

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![Diagram](image_url)
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![Diagram](diagram.png)
however extending this result leads directly to:

**Theorem:**

The LTL model checking question is undecidable for RQCPS that are well-queueing, eager and imbalanced. This already holds for a weak fragment of action LTL.

couple independent events by LTL interpreted on interleaving

\[ \text{run} \models \varphi \text{ iff actions sync'ed} \]
Theorem:
The LTL model checking question is undecidable for RQCPS that are well-queueing, eager and imbalanced. This already holds for a weak fragment of action LTL.

However extending this result leads directly to:

- Couple independent events by LTL interpreted on interleaving
- Leave interleaving semantics behind, look at rungraphs!
HRG for 1-stack reducible eager iRCPS

- **guess** “correct” labelling (transition rule \( \delta \in \Delta_p \) f.e. process \( p \))
- rulewidth bounded by \( \mathcal{O}(\#\text{procs.}) \)
- correctness of labelling “local”: express by MSO formula \( \varphi_{\text{label}} \)
- model **check** in addition: \( \varphi' \equiv \varphi \land \varphi_{\text{label}} \)
The MSO model checking question for rungraphs of RCPS that are eager, imbalanced and for which all runs can be reordered such that they are “1-stack reducible” is decidable.

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- cannot decide if for a given RCPS all runs can be reordered into a 1-stack one

- but covers important practical examples (e.g., master-worker protocols etc.)

- semantic boundedness versus syntactic description
MSO Model Checking RCPS

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Conjecture:

Bounded treewidth of the underlying rungraphs is the most general form of semantic boundedness condition for which MSO model checking is decidable.

- can generalize other boundedness conditions (e.g., bounded phases for RCPS, lock graphs, etc.)

- how to derive algorithms that are usable in practice

- how to derive syntactic/decidable restrictions

- what about other graph boundedness measures (cliquewidth, vertex cover, bounded pathwidth, ... )
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Treewidth Boundedness

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- what about other graph boundedness measures (cliquewidth, vertex cover, bounded pathwidth, ...)
• [Madhusudan et al.] LTL/MSO model checking for MSC-graphs

• [Parlato et al.’08] use tree-decomposition to derive new model checking algorithms (bounded phases...)

• MSO-definable temporal logics [Kuske/Gastin’10] and verification on partial orders of RCPS [Bollig et al.’11]

• partial-order versions of LTL (no consensus)

• novel boundedness conditions for asynchronous systems [LaTorre/Napoli’11] [Bouajjani/Emmi’12] etc.

• Autowrite tool [Durand/Courcelle]

• graph-rewriting based transition systems
Final Summary

▷ leave behind interleaving semantics

▷ focus rungraphs

▷ describe generation of rungraphs by graph grammars

▷ do model checking on rungraphs

▷ known connection HRG-treewidth-MSO allows to derive decidability of model checking