Safety Verification of Communicating One-Counter Machines

A. Heußner — Univ. of Bamberg
T. Le Gall — CEA Saclay
G. Sutre — LaBRI Bordeaux

FSTTCS 2012, Hyderabad

Agenda

- ➪ Motivating Example: Sliding Window Protocol
- ➪ Systems of Communicating One-Counter Machines and their Topology Parametrized Reachability Problem
- ➪ Main Theorem: Proof of “Only If” Direction
- ➪ Main Theorem: Proof of “If” Direction
- ➪ Related/On-going/Future Work

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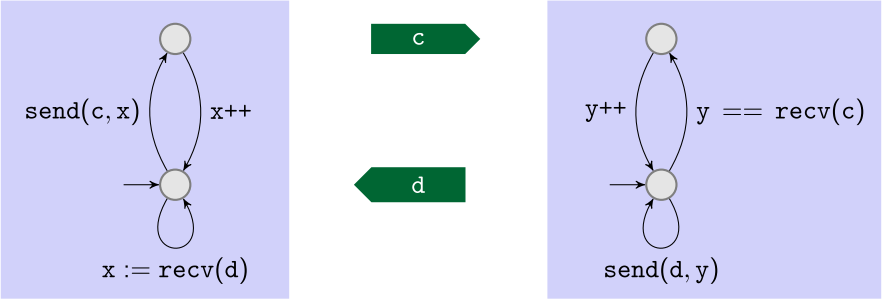
Example Sliding Window Protocol



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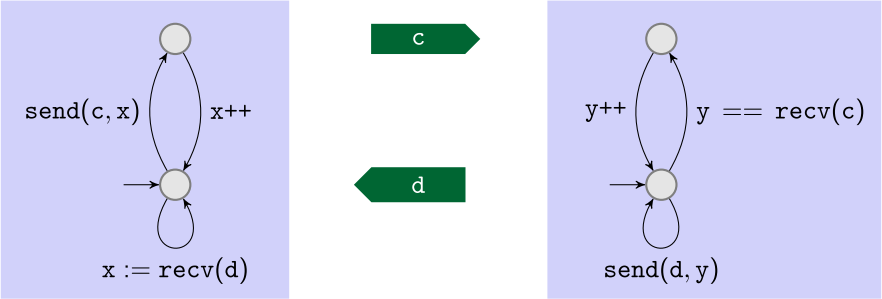

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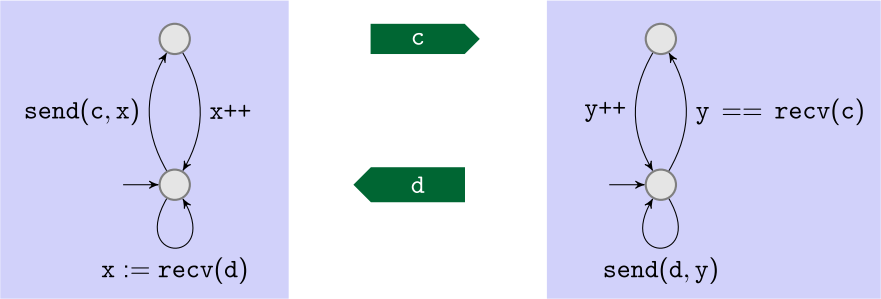

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- Asynchronous communication via perfect channels (c, d)
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  - receive and test/overwrite the counter
- Sources of infinity: local counters, message alphabet, channel length

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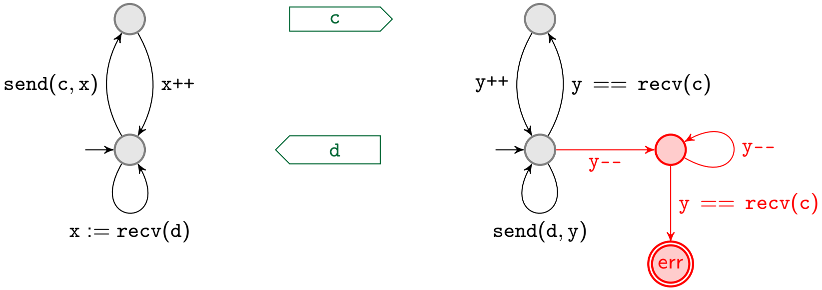

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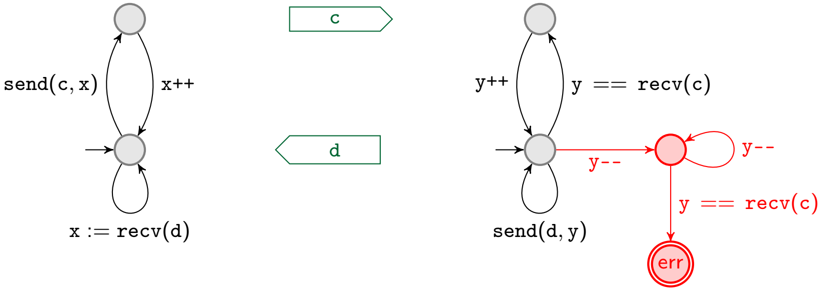
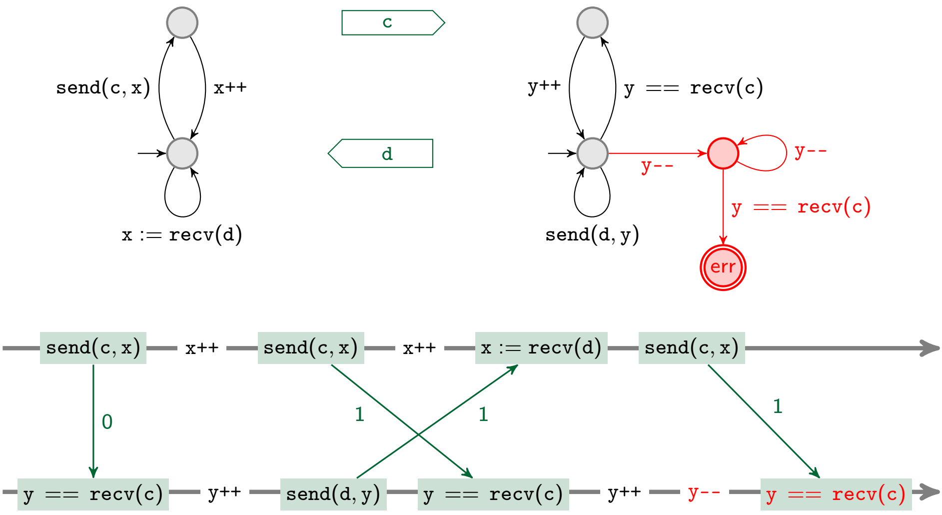


More formal goal:

Check that err is not reachable

An Erroneous Execution



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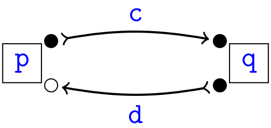
Communication Topologies



Definition:

A topology is $\mathcal{T} = \langle P, C, \mathrm{src}, \mathrm{dst} \rangle$ where

- ➪ P : finite set of processes
- ➪ C : finite set of channels
- ➪ $\mathrm{src}, \mathrm{dst} : C \to P \times \{\bullet, \circ\}$

Communication types

- $\bullet$ strong : standard CFSM-style communication ($==$)
- $\circ$ weak : counter lost by communication ($:=$)

Communicating One-Counter Machines

Definition:

A communicating one-counter machine is $\langle S, I, F, A, \Delta \rangle$ where

- S : finite set of states
- $I, F \subseteq S$: initial and final states
- A : finite set of actions
- $\Delta \subseteq S \times A \times S$: finite set of transition rules

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Definition:

A system of comm. one-counter machines is $\langle \mathcal{T}, (\mathcal{M}^p)_{p \in P} \rangle$ where

- $\mathcal{T}$: topology
- $\mathcal{M}^p$: communicating one-counter machine

SC1CM Semantics: Configurations

Recall:

A SC1CM is $\langle \mathcal{T}, (\mathcal{M}^p)_{p\in P} \rangle$ where $\mathcal{M}^p = \langle S^p, I^p, F^p, A^p, \Delta^p \rangle$

$$\text{A configuration is} \quad \big(\underset{\textstyle s}{\overset{\prod_{p\in P} S^p}{\Cup}} \; , \; \underset{\textstyle x}{\overset{\mathbb{N}^P}{\Cup}} \; , \; \underset{\textstyle w}{\overset{(\mathbb{N}^*)^C}{\Cup}} \big)$$

$$\begin{array}{rcl} \textit{initial} & \overset{\text{def}}{\Longleftrightarrow} & s^p \in I^p \\ \textit{final} & \overset{\text{def}}{\Longleftrightarrow} & s^p \in F^p \end{array} \quad \wedge \quad x = \mathbf{0} \quad \wedge \quad w = \varepsilon$$

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The transition relation $(s, x, w) \xrightarrow{a} (s', x', w')$ is defined by

- exactly one process moves
- counter actions behave as expected

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$$
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- Note: $\circ$ can be simulated by $\bullet$

Parametrized Reachability Problem

Definition:

Given a topology $\mathcal{T}$, the decision problem $\textsc{Rp-Sc1cm}(\mathcal{T})$ is

Input: a system of communicating one-counter machines $\mathcal{S}$ with topology $\mathcal{T}$

Output: whether there exists a full run in $[\![\mathcal{S}]\!]$

A run $(s, x, w) \xrightarrow{*} (s', x', w')$ is full when $\begin{cases} (s, x, w) & \text{is initial} \\ (s', x', w') & \text{is final} \end{cases}$

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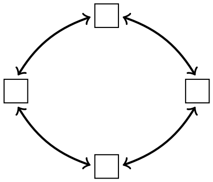
Goal:

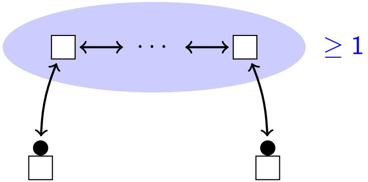
Characterize the topologies $\mathcal{T}$ where $\textsc{Rp-Sc1cm}(\mathcal{T})$ is decidable.

Main Result

Simple Undirected Cycle



Simple Undirected Shunt



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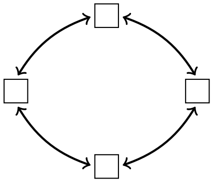

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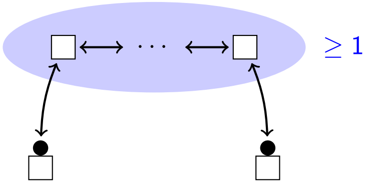

Theorem:

$\textsc{Rp-Sc1cm}(\mathcal{T})$ is decidable iff $\mathcal{T}$ is cycle-free and shunt-free

- cycle-free: no simple undirected cycle
- shunt-free: no simple undirected shunt

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Reduce known Undecidability Results

Idea:

Simulation of communicating finite-state machines (CFSM)

- ➪ encode finite message exchange in exchange of counters
- ➪ reduce undecidability of reachability on cyclic architectures ↯

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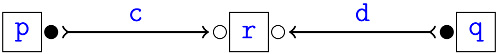
Simulation of two-counters Minsky machines



➪ reduce undecidability of reachability on simple shunt ↯

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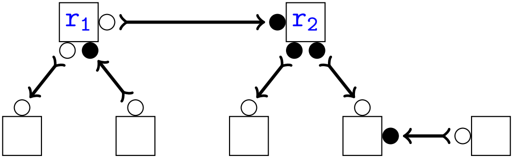
Cycle-free & Shunt-free Topologies

Form of topologies that are weakly-connected, cycle-free and shunt-free



- Two "roots" $\texttt{r}_1 \rightarrowtail \texttt{r}_2$
- Every simple undirected path from $\{\texttt{r}_1, \texttt{r}_2\}$ to $p \notin \{\texttt{r}_1, \texttt{r}_2\}$ ends with $\cdots \longleftrightarrow\!\circ\, p$

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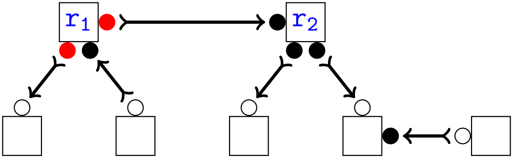

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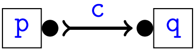

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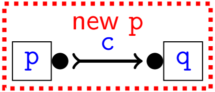

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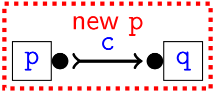

Idea:

Intersect reachability relations of p and q between synchronizations

$$\cdots \xrightarrow{\mathtt{c}\,!\,\mathtt{c}\,?} \underbrace{(s, x) \xrightarrow{*} (t, y)}_{\chi_{s,t}} \xrightarrow{\mathtt{c}\,!\,\mathtt{c}\,?} \cdots$$

$(s^{\mathtt{p}}, s^{\mathtt{q}})$

$(t^{\mathtt{p}}, t^{\mathtt{q}})$

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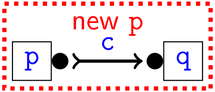

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(s^{p}, s^{q}) (t^{p}, t^{q})

$$\chi_{s,t}(x, y) \;=\; (s^{p}, x) \xrightarrow{*}_{p} (t^{p}, y) \;\wedge\; (s^{q}, x) \xrightarrow{*}_{q} (t^{q}, y) \;\in\; Presb_2$$

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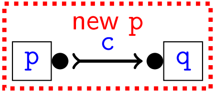

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Issue:

Reachability is undecidable ↯ for the class of one-counter machines with Presburger-definable updates

One-Counter Reachability Relations

Fix two distinguished Presburger variables $\mathtt{x}$ and $\mathtt{y}$

The class of one-counter Presburger predicates is generated by

$$\psi \quad ::= \quad \varphi(\mathtt{x}) \mid \varphi(\mathtt{y}) \mid \varphi(\mathtt{x}-\mathtt{y}) \mid \varphi(\mathtt{y}-\mathtt{x}) \mid \psi \wedge \psi \mid \psi \vee \psi \mid \mathtt{tt} \mid \mathtt{ff}$$

where φ ranges over unary Presburger predicates

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Theorem:

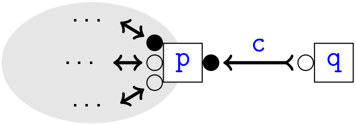
For every binary relation $R \subseteq \mathbb{N} \times \mathbb{N}$, the two following assertions are equivalent:

i) $R = \{(x, y) \mid (s, x) \xrightarrow{*} (t, y)\}$ for some one-counter machine

ii) $R = [\![\psi]\!]$ for some one-counter Presburger predicate ψ

➪ $\chi_{s,t}(x, y)$ can be translated into a one-counter machine ✓

Merging Leaf Processes



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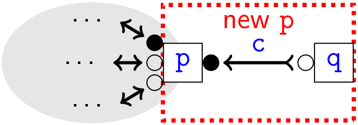

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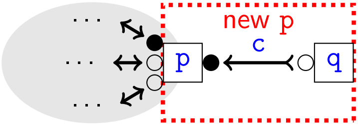

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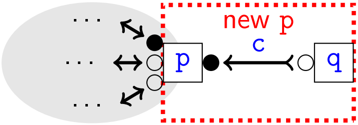

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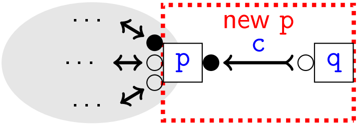

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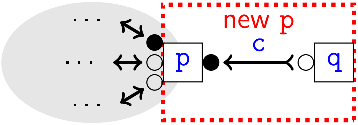

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- Rules $(s^{p}, c\,?, t^{p})$ of p become $((s^{p}, s^{q}), \mathtt{test}(\varphi), (t^{p}, t^{q}))$ where

$$\varphi \quad = \quad \exists u \exists z \cdot (u, c\,!, t^{q}) \in \Delta^{q} \ \wedge \ (s^{q}, z) \xrightarrow{*}_{q} (u, x)$$

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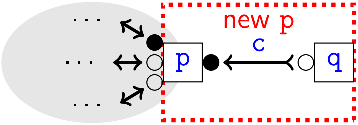

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- Use Presburger-definability of *post** for one-counter machines

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- ➪ Complete characterization of decidability in terms of topologies
- ➪ Technical (side-)result:
  reachability relations of one-counter machines fall in "good" fragment of Presburger arithmetics

Related Works

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  [Heußner/Leroux/Muscholl/Sutre '10]
- decidable restrictions of multi-counter machines,
  model checking register machines,...

Decidability of Eager Reachability

Definition:

A full run ρ is eager if matching $(c\,!, c\,?)$ pairs are consecutive in ρ

If $\mathcal{T}$ is cycle-free,

- ➪ Full runs can be re-ordered into eager ones
- ➪ $\textsc{Rp-Sc1cm-Eager}(\mathcal{T})$ is decidable iff $\mathcal{T}$ is shunt-free

Proposition:

If $\mathcal{T}$ is strongly connected, then $\textsc{Rp-Sc1cm-Eager}(\mathcal{T})$ is decidable iff $\mathcal{T}$ contains at most two processes

Open: full characterization of decidable topologies (for eager reachability)

Perspectives

Complexity of RP-SC1CM for decidable topologies

- ➪ At least PSPACE-hard

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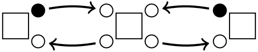
Lossy channel Communicating One-Counter Machines

➪ Undecidable for  using acknowledgments

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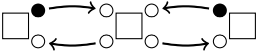
➪ Undecidable for  using acknowledgments

Extension from counters to stacks (i.e., send/receive stack)

Conjecture:

Rp-ScPDM($\mathcal{T}$) is decidable iff $\mathcal{T}$ is cycle-free and shunt-free